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**Mathematics: applications and interpretation**  
**Higher level**  
**Paper 1**

Monday 1 November 2021 (afternoon)

Candidate session number

2 hours

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.



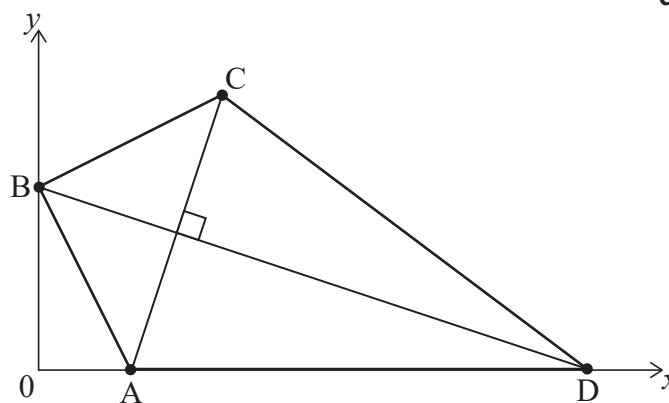
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Dilara is designing a kite ABCD on a set of coordinate axes in which one unit represents 10 cm.

The coordinates of A, B and C are (2, 0), (0, 4) and (4, 6) respectively. Point D lies on the  $x$ -axis. [AC] is perpendicular to [BD]. This information is shown in the following diagram.

diagram not to scale



- (a) Find the gradient of the line through A and C. [2]
- (b) Write down the gradient of the line through B and D. [1]
- (c) Find the equation of the line through B and D. Give your answer in the form  $ax + by + d = 0$ , where  $a$ ,  $b$  and  $d$  are integers. [2]
- (d) Write down the  $x$ -coordinate of point D. [1]

(This question continues on the following page)



(Question 1 continued)

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28EP03

Turn over

2. [Maximum mark: 5]

Inspectors are investigating the carbon dioxide emissions of a power plant. Let  $R$  be the rate, in tonnes per hour, at which carbon dioxide is being emitted and  $t$  be the time in hours since the inspection began.

When  $R$  is plotted against  $t$ , the total amount of carbon dioxide produced is represented by the area between the graph and the horizontal  $t$ -axis.

The rate,  $R$ , is measured over the course of two hours. The results are shown in the following table.

$t$	0	0.4	0.8	1.2	1.6	2
$R$	30	50	60	40	20	50

- (a) Use the trapezoidal rule with an interval width of 0.4 to estimate the total amount of carbon dioxide emitted during these two hours. [3]

The real amount of carbon dioxide emitted during these two hours was 72 tonnes.

- (b) Find the percentage error of the estimate found in part (a). [2]

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3. [Maximum mark: 7]

Let the function  $h(x)$  represent the height in centimetres of a cylindrical tin can with diameter  $x$  cm.

$$h(x) = \frac{640}{x^2} + 0.5 \text{ for } 4 \leq x \leq 14.$$

(a) Find the range of  $h$ . [3]

The function  $h^{-1}$  is the inverse function of  $h$ .

- (b) (i) Find  $h^{-1}(10)$ .
- (ii) In the context of the question, interpret your answer to part (b)(i).
- (iii) Write down the range of  $h^{-1}$ . [4]

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28EP05

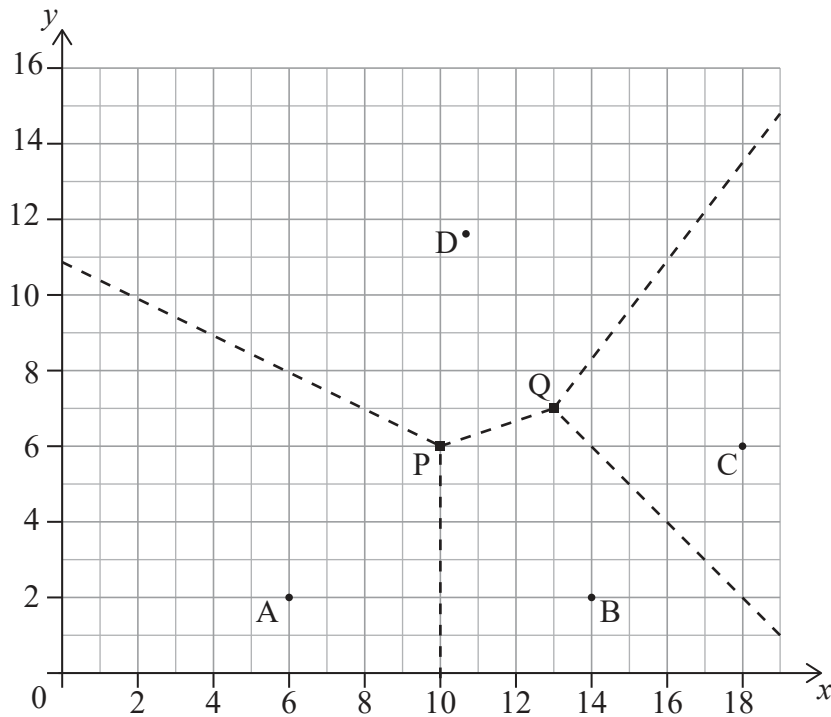
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4. [Maximum mark: 6]

There are four stations used by the fire wardens in a national forest.

On the following Voronoi diagram, the coordinates of the stations are  $A(6, 2)$ ,  $B(14, 2)$ ,  $C(18, 6)$  and  $D(10.8, 11.6)$  where distances are measured in kilometres.

The dotted lines represent the boundaries of the regions patrolled by the fire warden at each station. The boundaries meet at  $P(10, 6)$  and  $Q(13, 7)$ .



To reduce the areas of the regions that the fire wardens patrol, a new station is to be built within the quadrilateral  $ABCD$ . The new station will be located so that it is as far as possible from the nearest existing station.

- (a) Show that the new station should be built at  $P$ . [3]

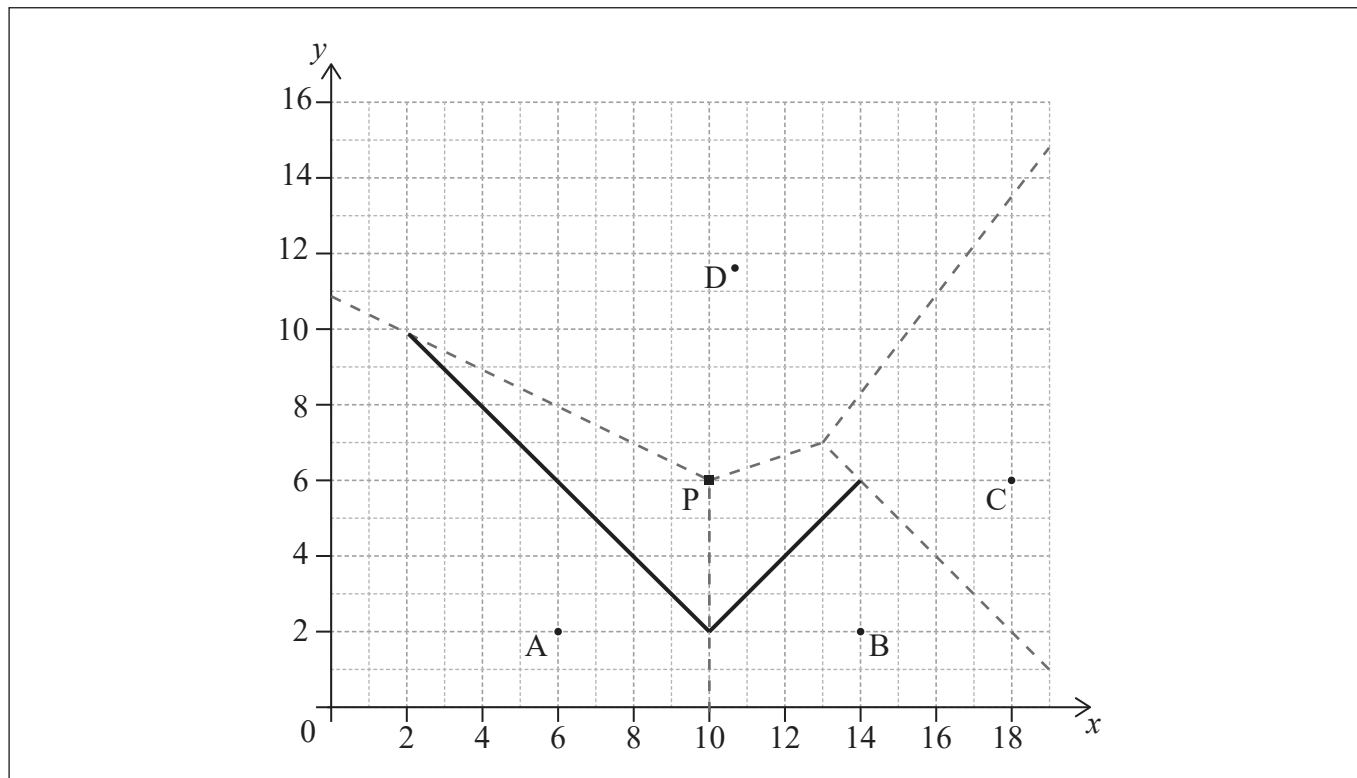
The Voronoi diagram is to be updated to include the region around the new station at  $P$ . The edges defined by the perpendicular bisectors of  $[AP]$  and  $[BP]$  have been added to the following diagram.

- (b) (i) Write down the equation of the perpendicular bisector of  $[PC]$ .  
 (ii) Hence draw the missing boundaries of the region around  $P$  on the following diagram. [3]

(This question continues on the following page)



(Question 4 continued)



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5. [Maximum mark: 9]

**In this question, give all answers correct to 2 decimal places.**

Raul and Rosy want to buy a new house and they need a loan of 170 000 Australian dollars (AUD) from a bank. The loan is for 30 years and the annual interest rate for the loan is 3.8%, compounded monthly. They will pay the loan in fixed monthly instalments at the end of each month.

- (a) Find the amount they will pay the bank each month. [3]
  
- (b) (i) Find the amount Raul and Rosy will still owe the bank at the end of the first 10 years.
  
- (ii) Using your answers to parts (a) and (b)(i), calculate how much interest they will have paid in total during the first 10 years. [6]

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6. [Maximum mark: 5]

An infinite geometric sequence, with terms  $u_n$ , is such that  $u_1 = 2$  and  $\sum_{k=1}^{\infty} u_k = 10$ .

(a) Find the common ratio,  $r$ , for the sequence. [2]

(b) Find the least value of  $n$  such that  $u_n < \frac{1}{2}$ . [3]

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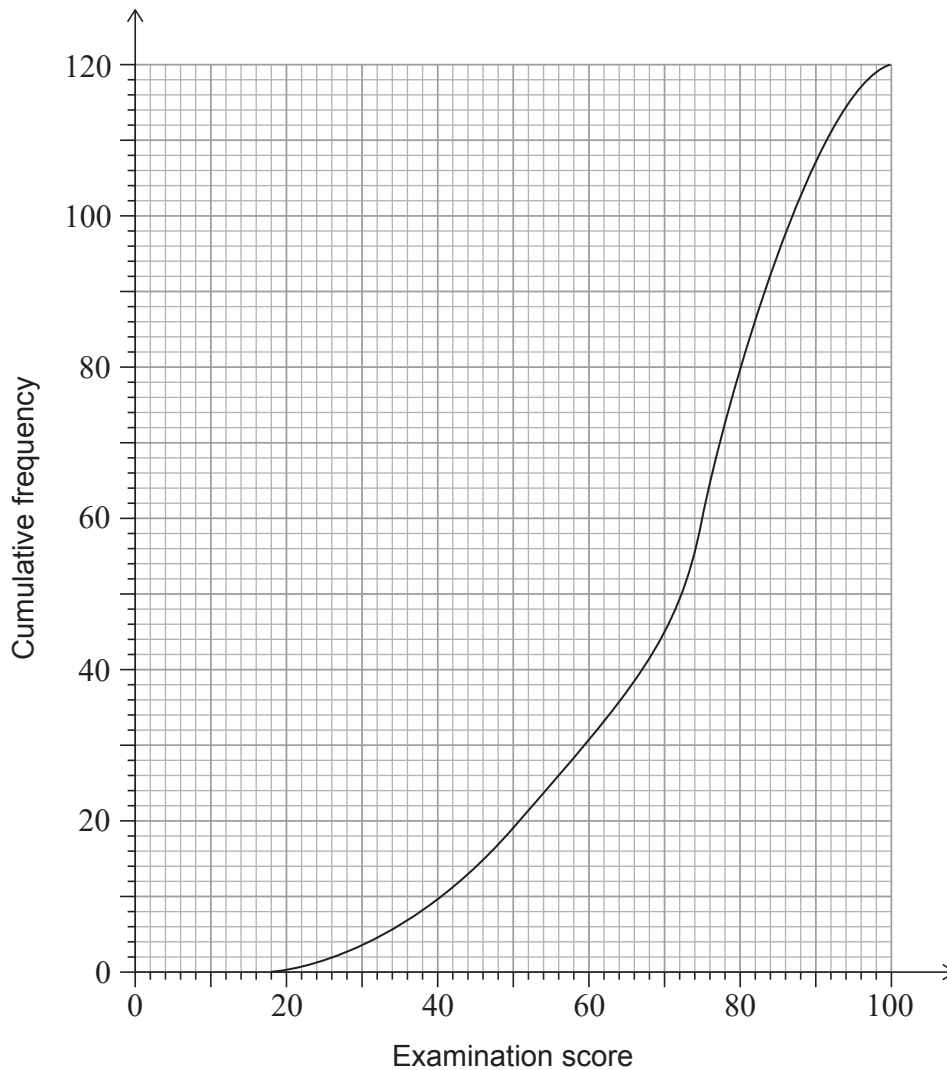
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7. [Maximum mark: 8]

A group of 120 students sat a history exam. The cumulative frequency graph shows the scores obtained by the students.



(a) Find the median of the scores obtained.

[1]

The students were awarded a grade from 1 to 5, depending on the score obtained in the exam. The number of students receiving each grade is shown in the following table.

<b>Grade</b>	1	2	3	4	5
<b>Number of students</b>	6	13	26	$a$	$b$

(b) Find an expression for  $a$  in terms of  $b$ .

[2]

(This question continues on the following page)



28EP10

**(Question 7 continued)**

(c) The mean grade for these students is 3.65.

(i) Find the number of students who obtained a grade 5.

(ii) Find the minimum score needed to obtain a grade 5.

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28EP11

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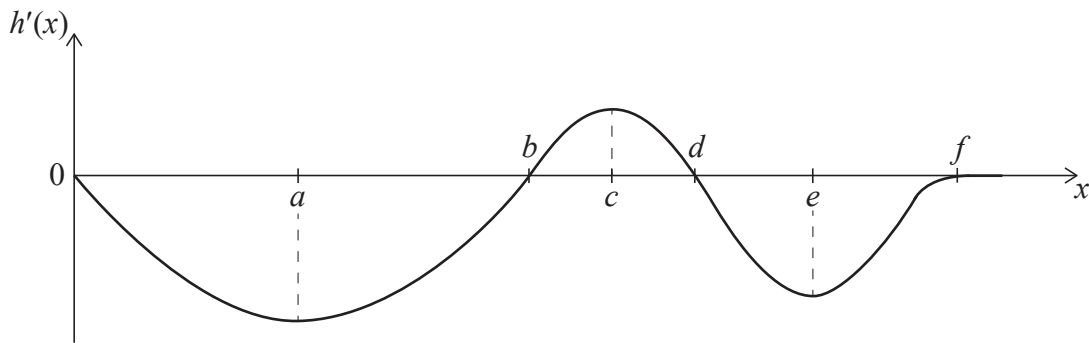
8. [Maximum mark: 5]

Juri skis from the top of a hill to a finishing point at the bottom of the hill. She takes the shortest route, heading directly to the finishing point (F).



Let  $h(x)$  define the height of the hill above F at a horizontal distance  $x$  from the starting point at the top of the hill.

The graph of the **derivative** of  $h(x)$  is shown below. The graph of  $h'(x)$  has local minima and maxima when  $x$  is equal to  $a$ ,  $c$  and  $e$ . The graph of  $h'(x)$  intersects the  $x$ -axis when  $x$  is equal to  $b$ ,  $d$ , and  $f$ .



- (a) (i) Identify the  $x$  value of the point where  $|h'(x)|$  has its maximum value.
- (ii) Interpret this point in the given context.

[2]

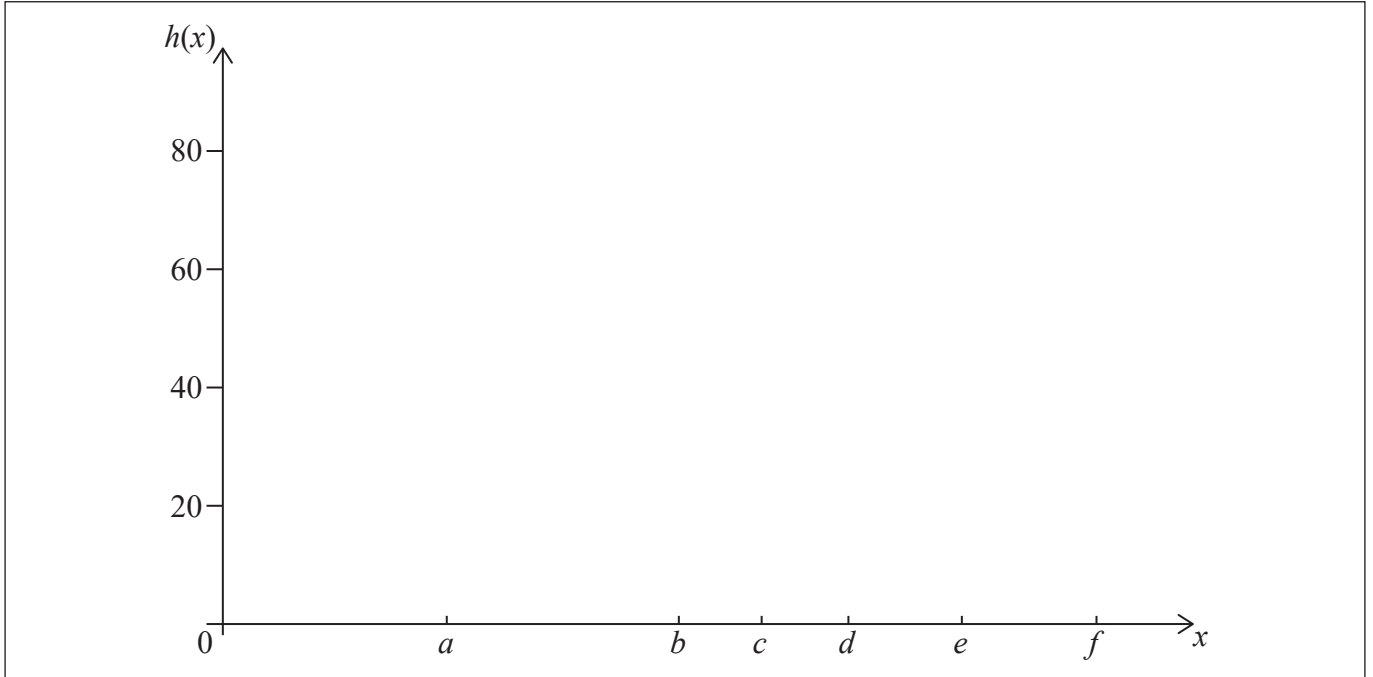
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**(Question 8 continued)**

Juri starts at a height of 60 metres and finishes at F, where  $x = f$ .

- (b) Sketch a possible diagram of the hill on the following pair of coordinate axes. [3]



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28EP13

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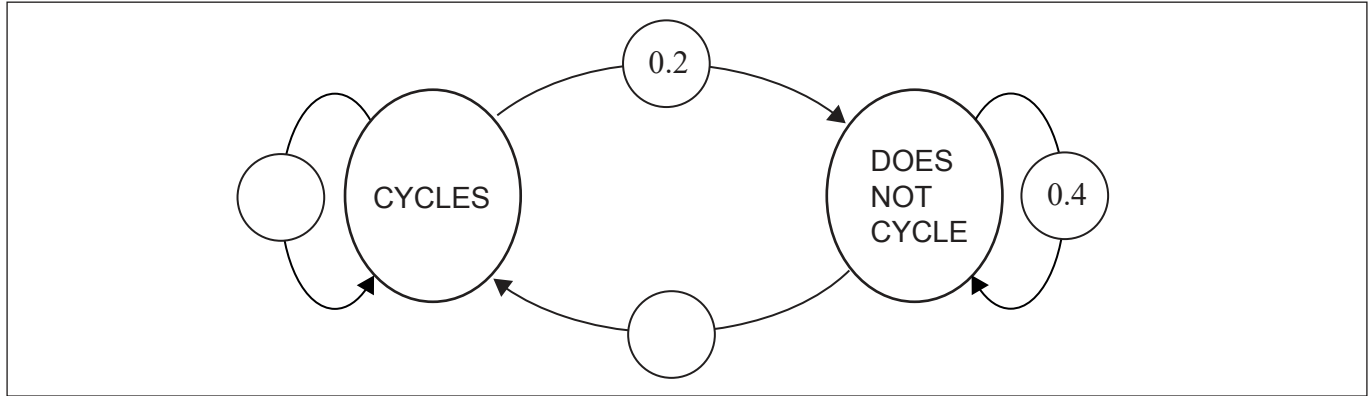


28EP14

9. [Maximum mark: 5]

Katie likes to cycle to work as much as possible. If Katie cycles to work one day then she has a probability of 0.2 of not cycling to work on the next work day. If she does not cycle to work one day then she has a probability of 0.4 of not cycling to work on the next work day.

(a) Complete the following transition diagram to represent this information. [2]



Katie works for 180 days in a year.

(b) Find the probability that Katie cycles to work on her final working day of the year. [3]

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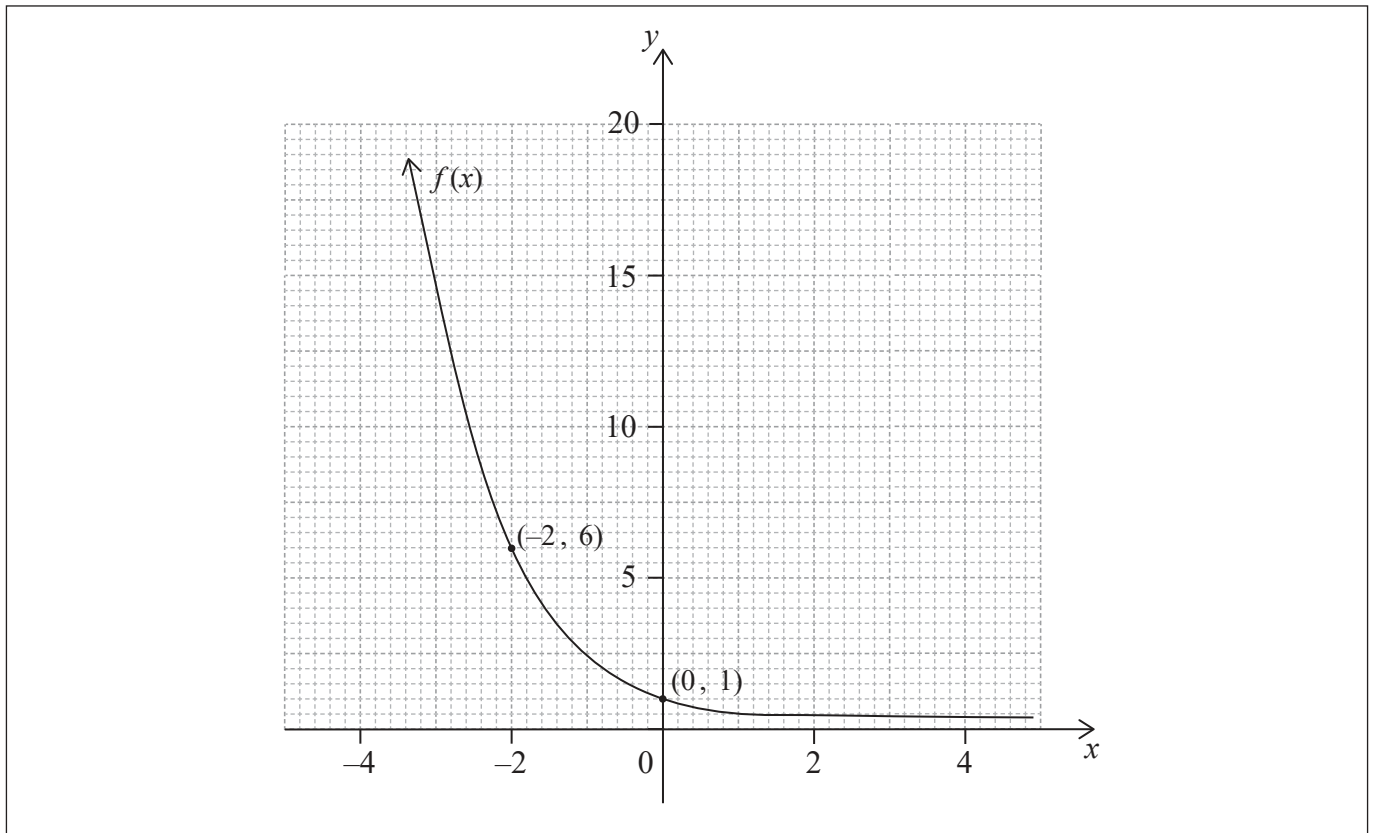
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10. [Maximum mark: 4]

The graph of  $y = f(x)$  is given on the following set of axes. The graph passes through the points  $(-2, 6)$  and  $(0, 1)$ , and has a horizontal asymptote at  $y = 0$ .



Let  $g(x) = 2f(x - 2) + 4$ .

- (a) Find  $g(0)$ . [2]
- (b) On the same set of axes draw the graph of  $y = g(x)$ , showing any intercepts and asymptotes. [2]

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(Question 10 continued)

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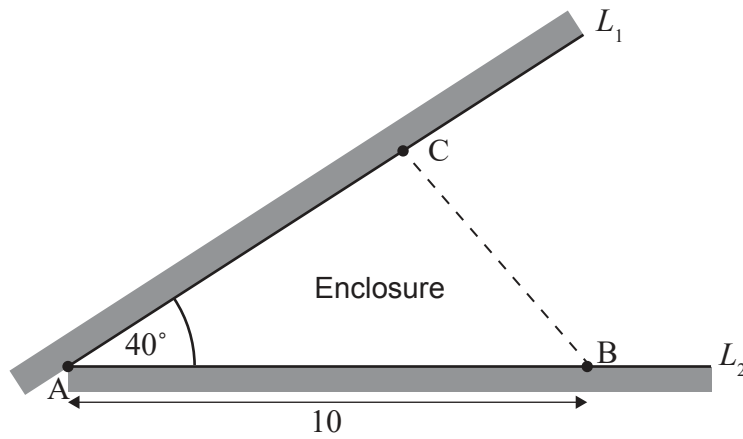
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11. [Maximum mark: 6]

The following diagram shows a corner of a field bounded by two walls defined by lines  $L_1$  and  $L_2$ . The walls meet at a point A, making an angle of  $40^\circ$ .

Farmer Nate has 7 m of fencing to make a triangular enclosure for his sheep. One end of the fence is positioned at a point B on  $L_2$ , 10 m from A. The other end of the fence will be positioned at some point C on  $L_1$ , as shown on the diagram.

diagram not to scale



He wants the enclosure to take up as little of the current field as possible.

Find the minimum possible area of the triangular enclosure ABC.

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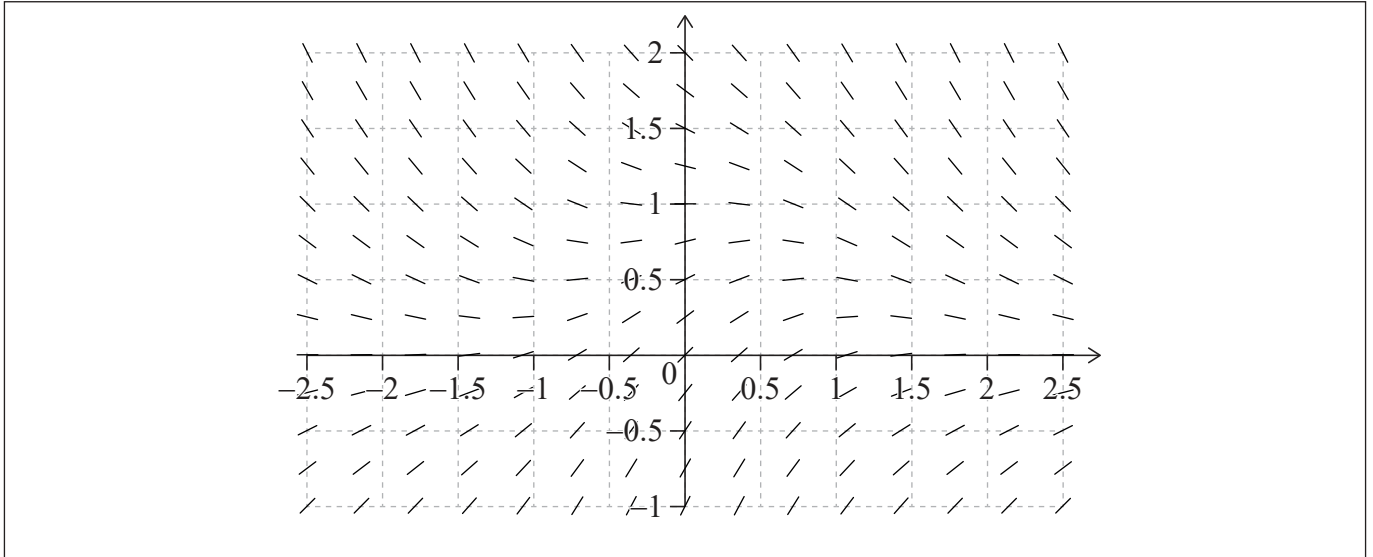


13. [Maximum mark: 7]

The slope field for the differential equation  $\frac{dy}{dx} = e^{-x^2} - y$  is shown in the following two graphs.

(a) Calculate the value of  $\frac{dy}{dx}$  at the point (0, 1). [1]

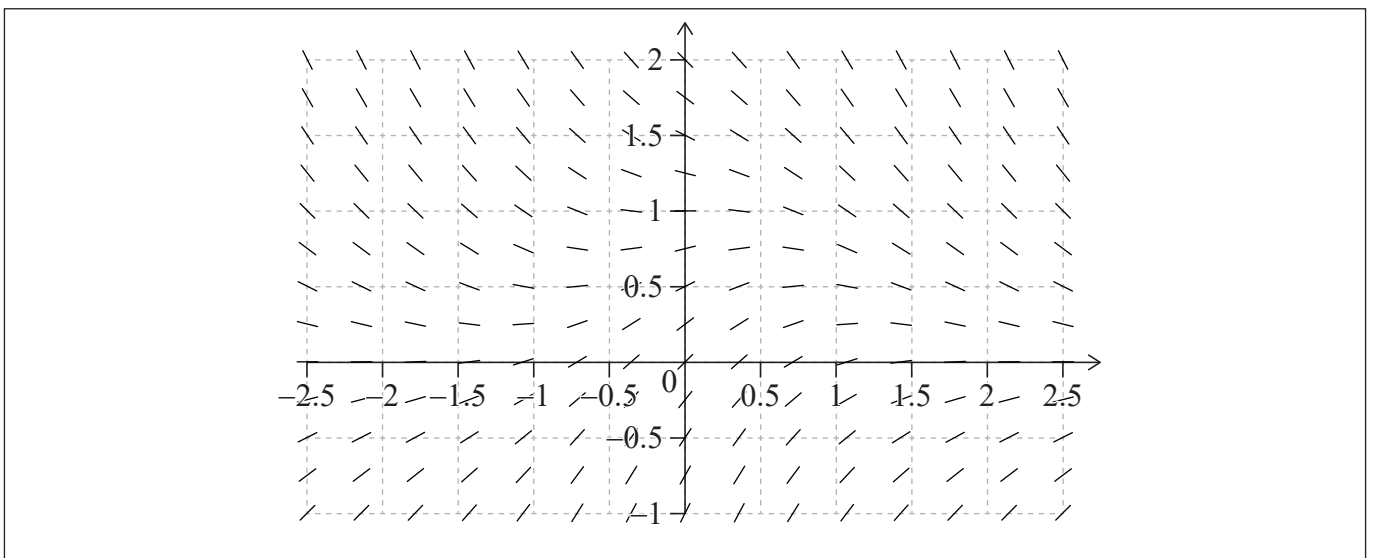
(b) Sketch, on the first graph, a curve that represents the points where  $\frac{dy}{dx} = 0$ . [2]



(c) On the second graph,

(i) sketch the solution curve that passes through the point (0, 0).

(ii) sketch the solution curve that passes through the point (0, 0.75). [4]



(This question continues on the following page)





14. [Maximum mark: 7]

On Paul's farm, potatoes are packed in sacks labelled 50 kg. The weights of the sacks of potatoes can be modelled by a normal distribution with mean weight 49.8 kg and standard deviation 0.9 kg.

- (a) Find the probability that a sack is under its labelled weight. [2]
- (b) Find the lower quartile of the weights of the sacks of potatoes. [2]

The sacks of potatoes are transported in crates. There are 10 sacks in each crate and the weights of the sacks of potatoes are independent of each other.

- (c) Find the probability that the total weight of the sacks of potatoes in a crate exceeds 500 kg. [3]

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16. [Maximum mark: 9]

A ship S is travelling with a constant velocity,  $\mathbf{v}$ , measured in kilometres per hour, where

$$\mathbf{v} = \begin{pmatrix} -12 \\ 15 \end{pmatrix}.$$

At time  $t = 0$  the ship is at a point A(300, 100) relative to an origin O, where distances are measured in kilometres.

(a) Find the position vector  $\overrightarrow{OS}$  of the ship at time  $t$  hours. [1]

A lighthouse is located at a point (129, 283).

(b) Find the value of  $t$  when the ship will be closest to the lighthouse. [6]

An alarm will sound if the ship travels within 20 kilometres of the lighthouse.

(c) State whether the alarm will sound. Give a reason for your answer. [2]

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